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SEEPAGE ANALYSIS BASED ON THE UNIFIED UNSATURATED SOIL THEORY

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Abstract The normal use of the finite element method in the analysis of earth and rock-fill dams involves troublesome modifications of the finite element mesh In the present paper it is pointed out that in problems of steady seepage it is not necessary to determine in the iteration process the entire free surface, but only the elevation of the release point It is shown by several examples that the proposed method can simplify the seepage analysis to a certain degree, and also give satisfactory results

Introduction

Seepage analysis plays an important role in designs of hydraulic structures, such as earth and rock-fill dams It can significantly affect the safety and cost of the structures Numerical methods are usually adopted to solve the seepage problems encountered in actual practice Among these approaches, finite element method (FEM) is widely accepted owing to its extensive adaptation to complicated boundaries, anisotropy, inhomogeneous material and three-dimensional problems Presently, FEM techniques related to various field problems have progressed rapidly Many general commercial software usually have modules for heat conduction [1], which is similar to seepage phenomenon However, seepage through pervious earth and rock-fill dam has a phreatic surface, or free surface The release point where the free surface intersects the downstream batter is higher than the elevation of downstream water level This problem is more complicated for FEM used for steady heat conduction However, there has been a lot of papers aiming to solve this problem

Zienkiewicz et al firstly used FEM to solve confined seepage problems [2], then Taylor et al used FEM to determine the free surface by modifying the mesh which only represents the saturated soil domain [3] Until now, most of the methods referenced determine the free surface by iteration of a series of points that form the surface

There are two main categories of finite element approaches for solution of free surface of seepage problems The first group of these approaches requires modification of the mesh at each iteration step, which needs great computational effort, owing to the intrinsic geometry of the mesh [4] The second group has a mesh of constant geometry [5-8], while a suitable pressure-permeability law might be needed to identify the saturated and unsaturated zone, resulting in adjustment of the material parameters during solution process

Cividini and Gioda presented an approximate solution of the free surface of transient seepage problems without modification of the mesh [9] The free surface is represented by a series of segments that coincide with sides of the elements of the mesh Lacy and Prevost employed a penalty procedure to locate the free surface by controlling the fluid pressure to a small specified negative value in the dry region [10] Their method needed refinement of the mesh

This paper presents a new approach by only iterating the elevation of the release point, and the free surface can be directly determined from the results of pore pressure field, so that the seepage analysis is simplified to a certain degree and the accuracy is also satisfied Based on the new approach, ordinary seepage analysis can also be carried out by employing any general commercial software in hand

Description of the approach

Considering the inhomogeneous, anisotropy medium which obeys Darcy's law, with the geometrical coordinate axis coincide with the seepage one, the controlling equations and boundary conditions of the three dimensional steady seepage problem are described as [11]

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial H}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial H}{\partial z}\right)+w=0 \tag{1}$$

in domain Ω

$$H(x,y,z)=\overline{H}(x,y,z) \tag{2}$$

on border S_1

$$k_x\frac{\partial H}{\partial x}\cos(n,x)+k_y\frac{\partial H}{\partial y}\cos(n,y)+k_z\frac{\partial H}{\partial z}\cos(n,z)=\overline{Q} \tag{3}$$

on S_2 and S_3

$$H(x,y,z)=z(x,y) \tag{4}$$

on S_3 and S_4

where, z is the coordinate of elevation, H is the total head at any point in a domain denoted as Ω, k_x, k_y and k_z are the permeability coefficients in x, y and z directions respectively w is the water absorbed by or separated out from soil particles, $\overline{H}$ is the given total head, $\overline{Q}$ is the given permeability rate, S_1 is the boundary with given total head, S_2 is the boundary with given permeability rate, S_3 is the free surface, S_4 is the downstream batter below the release point n is the unit vector on upper normal direction at any point of S_2 and S_3

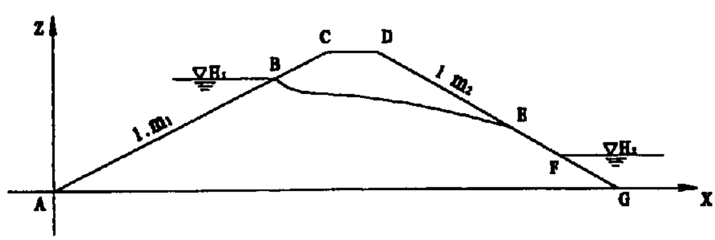

Fig 1 Steady seepage of earth and rock-fill dam

Referring to the steady seepage problem of earth and rock-fill dam in Fig 1, Ω is the seepage domain located in the range of ABEFGA AB, FG and the release section EF form the boundary S_1 with given total head AG and free surface BE form the impervious boundary S_2, and the pore pressure on BE and EF line is zero That is to say, on the free surface, conditions (3) and (4) should be satisfied simultaneously, while in (3), q is set to be zero However, the position of the free surface is unknown, it ought to be determined during the solving of the equations At present, iterative computing method is employed to determine the free surface Assuming the initial position of the free surface, then solving the ordinary boundary problem including Equations (1)-(3), the value of total head at any point in seepage area is thus calculated The analysis checks if condition (4) is satisfied in the free surface under a permissible tolerance If it is satisfied, the iteration is completed Otherwise, the position of the free surface should be modified, and the iteration has to be repeated until the controlling condition is satisfied

This method is simple and easily adapted in a certain sense However, if the assumed position of free surface is fairly far from the actual one, and if the finite element mesh can't be efficiently adjusted during iteration, there might generate singular elements and leads to serious errors, or even interrupt the calculation [11]

To avoid this difficulty, a fully determined boundary is used to replace the assumed free surface Also referring to Fig 1, the seepage area should be taken as ABCDEFGA, AB, FG and the release section EF form the boundary S_1 with given total head, AG and the crest configuration of the dam BCDE form the impervious boundary S_2 Within the downstream batter DG, DE is an impervious boundary, while EF is a boundary with zero pore pressure The elevation of the release point E is still unknown, which also needs to be determined by iteration However, by this new approach, the position that needs to be determined is not a series node which forms the entire free surface, but just the release point The free surface is determined by interpolation of the pore pressure field, so there involves no modification of mesh in the seepage area, which can make the initial discretion of mesh very flexible Only the mesh in the vicinity of the release point needs to be refined This new approach can largely facilitate the solving of the problem

The new approach assumes the initial elevation of the release point at first, then solves the problem including Equations (1)-(3) The total head field is thus calculated The intersection of the zero pore pressure contour and the downstream batter is compared with the assumed release point If the two points coincide with each other under a permissible tolerance, then the calculation is completed Otherwise, the elevation of the release point is modified, and the iteration is repeated until this controlling condition is satisfied

On summary, the key point of the new approach is that, the free surface is not regarded as a fully impervious boundary but just an approximate impervious one, on which the normal seepage velocity is almost, but not exactly, equals to zero However, the pore water pressure on the free surface is defined as zero, which is just the same as the ordinary approach The principle of the new approach is not contradicted to the concept of seepage Actually, seepage flow definitely takes place in soils above the free surface, but mostly it was explained to be merely induced by capillary force

It must be point out that, the linear Laplace equation (1) is not accurate in describing the steady seepage in unsaturated soil above the free surface A unified nonlinear model should be adopted to describe the seepage flow above and below the free surface, and the overall dam is considered as the possible seepage area instead of only the part of dam below the free surface

In unsaturated soil mechanics, there is a clear nonlinear relation between permeability coefficient k and pore pressure p as [12]

$$k_w = \frac{k_s}{1 + a\left(\frac{p}{\gamma}\right)^n} \tag{5}$$

where, k_w is permeability coefficient for unsaturated soil, k_s is permeability coefficient for saturated soil, γ is the specify gravity of water, a and n are constants However, the relation between k and total head H includes the effect of elevation head, it is convenient to use pore pressure as the unknown variable in equations (1)-(4) Let

$$p(x,y,z) = \gamma[H(x,y,z) - z(x,y)] \tag{6}$$

then (1)-(4) become

$$\frac{\partial}{\partial x}\left(k_x(p)\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y(p)\frac{\partial p}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z(p)\frac{\partial p}{\partial z}\right) + \gamma w = 0 \tag{1a}$$

in domain Ω

$$p(x,y,z) = \bar{p}(x,y,z) \tag{2a}$$

on border S_1

$$k_x\frac{\partial p}{\partial x}\cos(n,x) + k_y\frac{\partial p}{\partial y}\cos(n,y) + k_z\frac{\partial p}{\partial z}\cos(n,z) = \gamma\overline{Q} - \gamma k_z\cos(n,z) \tag{3a}$$

on S_2

$$p(x,y,z) = 0 \tag{4a}$$

on S_4

For an impervious boundary, (3a) becomes

$$k_x\frac{\partial p}{\partial x}\cos(n,x) + k_y\frac{\partial p}{\partial y}\cos(n,y) + k_z\frac{\partial p}{\partial z}\cos(n,z) = -\gamma k_z\cos(n,z) \tag{3b}$$

Except using pore pressure p as the unknown variable instead of using total head H, (1a), (2a) and (4a) are almost similar to (1), (2), and (4) The only obvious difference is that when using pore pressure as the unknown variable, the boundary seepage discharge in (3) has to be modified Even for an impervious boundary, there is a value that has to be input on S_2

Compare with the ordinary method, additional work is needed in preparing for the input data, but equation (1a) is a standard nonlinear seepage equation, with a determined boundary condition (2a) and (3b), it can be solved by many general commercial software Considering the advantage of speed, precision, convenience, and function of general commercial software, it is worth adopting this new approach

Naturally, the crest configuration BCDE has zero pore water pressure, since the soil particles are in contact with the atmosphere However, due to the randomness of the distribution of the pore between particles, at any location a short distance from the surface BCDE, it can be approximately regarded as impervious condition Since the earth and rock-fill dam usually has a large dimension, it is feasible for the new approach to employ the crest configuration BCDE as an impervious boundary

Examples and discussion

Case 1 Uniform earth and rock-fill dam

In order to verify the effectiveness of this new approach, seepage in a uniform earth and rock-fill dam shown in fig 1 was analyzed using general commercial software ABAQUS, which is usually used for heat conduction analysis Geometry of the homogeneous dam in Fig 1 is that, slope of upstream batter is 1 1 98, slope of downstream batter is 1 1 71 Width and elevation of the crest is 17 m and 45 m respectively The upstream elevation head is 40 m, the downstream elevation head is 0 m a and n in equation (5) are 0 15 and 6 respectively

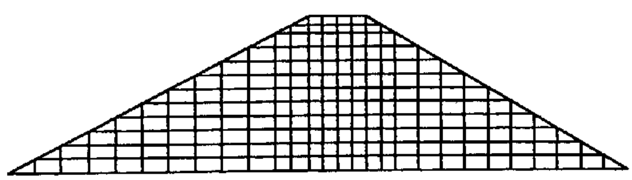

Fig 2 FEM meshes of a uniform earth and rock-fill dam

Chen provides the results of position of the free surface and distribution of pore water pressure, using ordinary iteration [13] With the new approach, the mesh (shown in fig 2) can be generated with great flexibility related to the actual position of the free surface The initial elevation of the release point can be assumed freely along the downstream batter For convenience, it can be assumed to be the downstream water level Table 1 lists the results of each step of iteration

Table 1 Results of iteration

Step of Iteration	Assumed elevation of release point (m)	Calculated elevation of release point (m)	Relative Tolerance (%)
1	0	34 56	200
2	17 28	20 98	19 3
3	19 13	20 14	5 1
4	19 64	19 64	0 0

The result of convergence is therefore 19 64 m

The calculation revealed the following phenomena

(1) If the assumed release point E is higher than the real one, the calculated zero pore pressure contour still intersects the downstream batter at point E, while in a part of DG below the point E, the calculated value of pore pressure is also zero This phenomenon is induced by the enforced boundary condition of given pore pressure in section FE on downstream batter In this case, the ordinary approach also has this kind of difficulty It will give a free surface which sticks up near the release point The reason is the same as that mentioned above To solve this problem, the iteration of the release point can be start out from the downstream water level, so that the assumed

elevation in each step of iteration will be kept lower than the actual value However, the convergent result might also be slightly higher than the actual one Hence the iteration must be kept in between the convergent result 19 64 m and the last iteration one 19 26 m, the lowest point which satisfies condition (4) is the actual release point After three more steps of iteration, it can be seen that 19 20 m is surely lower than the actual release point, while 19 26 m is probably higher than it The relative tolerance between these two values is just 0 3% So 19 26 m can be regarded as close to the elevation of release point Fig 3 shows the contours of pore pressure of this example In terms of its physical concept, the contour with zero pore pressure is the phreatic line These results coincided with those reported by Chen [13]

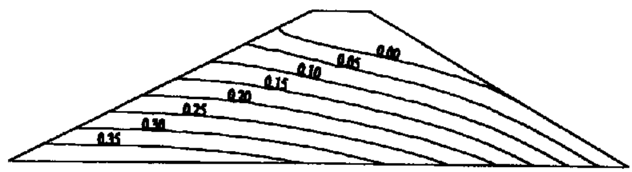

Fig 3 Distribution of pore pressure (unit 10^{-2}MPa)

(2) It can be obtained just from the calculation (Figure 3) that, the pore pressure in the unsaturated soil above the free surface is negative, and even though the impervious boundary are relaxed to BCDE, the calculated seepage rate above the free surface are really very small These results agreed with the qualitative explanation of the classical seepage theory

(3) If the assumed release point is not very far away from the actual one, the distribution of pore pressure have little difference from the actual one, so does the position of the phreatic line except the points near the release point Consequently, the iteration tolerance need not be too small With a relatively fine mesh in the vicinity of the possible release point, the modification of mesh is not needed during iteration

Case 2 Nonhomogeneous earth and rock-fill dam

The following is an example of a dam with permeable foundation and toe drain, quoted from Qian and Yin (1994), which is shown in Fig 4 The permeability coefficient of the foundation of sand layer is 125 times of the earth dam and blanket

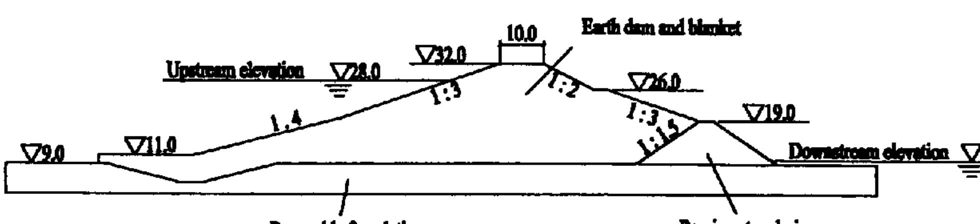

Fig 4 A nonhomogeneous earth and rock-fill dam with permeable foundation and pervious toe drain

In calculation, the dam, foundation and toe drain are all considered in the mesh The toe drain was given a large value of permeability coefficient (which is 10000 times larger than the permeability coefficient of the dam) According to the new approach, the initial elevation of the release point is assumed as downstream water level At the first iteration, the calculated zero pore pressure contour intersects with downstream batter of the drain prism also at this elevation So that no more iteration is needed Fig 5 illustrates the distribution of the total head and the position of the free surface, which show good agreement with Qian and Yin [14] In case of dam with core wall, results can also be directly obtained without iteration

From the calculation, it is shown that there exists no release section in the case when the dam has toe drain and the downstream water level is lower then the crest of the prism This result also agreed with ordinary observation

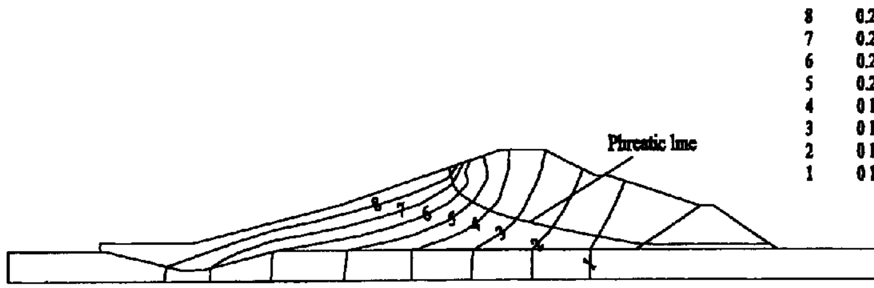

Fig 5 Distribution of the total head (unit 10 ^{2}MPa)

Conclusion

Based on the unified unsaturated soil theory, a simple approach to determine the position of free surface in steady seepage is developed in this paper In steady seepage of earth and rock-fill dam, the crest configuration can be regarded as impervious boundary Only the elevation of the release point must be iterated in order to calculate the phreatic line, instead of iterating the position of a series of nodes which form the phreatic line in conventional method Consequently, the initial mesh can be generated without any limitation, and the finite element mesh needs not be modified during calculation This can largely simplify the seepage analysis

Based on the concepts discussed in the above section, the analysis can be simplified, and most of the ordinary seepage analysis can be carried out by general commercial software, also the speed, precision, convenience, and function of general commercial software in field analysis

In the analysis of nonhomogeneous dam with toe drain or core wall, results can be obtained directly without iteration This is an important feature of the new approach

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