Evaluation of active earth pressure by the generalized method of slices

Zuyu Chen and Songmei Li

Abstract: The generalized method of slices, commonly used in slope stability analysis, can be extended to determine active earth pressures applied to various types of supports. The governing force and moment equilibrium equations are given. In a similar manner to slope stability analysis, the methods of optimization are used to define the critical slip surface that is associated with the maximum wall pressure. Examples show that the approaches give active earth pressures identical to the Rankine solution for gravity walls. For other types of support, such as anchored or struette walls, the earth pressure is determined by assigning appropriate locations of the point of application on the wall. It has been found that applying the restrictions of physical admissibility is more vital in earth pressure problems than in slope stability assessments.

Key words: earth pressure, limit equilibrium method, the method of slices, retaining walls.

Résumé : La méthode généralisée des tranches utilisée couramment dans l’analyse de stabilité des talus peut être élargie pour déterminer la poussée des terres s’appliquant sur différents types de soutènement. Les équations régissant l’équilibre des forces et des moments sont données. Comme dans l’analyse de stabilité des talus, les méthodes d’optimisation sont utilisées pour définir la surface critique de glissement qui est associée à la pression maximale sur le mur. Des exemples montrent que les approches donnent des poussées identiques à la solution de Rankine pour la solution des murs gravité. Pour les autres types de soutènement tels que les murs ancrés ou les murs étayés, la pression des terres est déterminée en assignant des localisations appropriées de points d’application sur le mur. L’on a trouvé que l’application des restrictions d’admissibilité physique est plus vitale dans les problèmes de pression des terres que dans les évaluations de stabilité des talus.

Mots clés : pression des terres, méthode d’équilibre limite, la méthode des tranches, murs de soutènement.

[Traduit par la Rédaction]

Introduction

The lateral supporting force provided by a soil retaining structure, commonly referred to as active earth pressure, varies with the flexibility of the wall. After a review of several case histories, Casagrande (1973) concluded that the active earth pressure on a wall with a cantilever or tie-back support may go far beyond the value calculated by the classical approach and may reach a magnitude close to the earth pressure at rest. It is well understood that a supported wall, such as a braced sheet, usually allows larger displacement, compared to a gravity wall. As a consequence, the pressure distribution on the wall presents a trapezoidal shape, compared to a triangular one for a gravity wall. The classical Rankine and Coulomb’s theories, which employ force equilibrium conditions only, could not allow for a rational consideration of the differences in pressure distribution on the wall.

The generalized methods of slices (Morgenstern and Price 1965; Janbu 1973; Spencer 1967) as used in slope stability analysis consider both force and moment equilibrium and thus have the potential of being extended to earth pressure calculations. It is suggested that a retaining wall is in fact a special slope with a vertical front face. The difference in the pressure distribution for different types of supports can be properly considered in the generalized methods by assigning different locations of the point of application of the lateral earth pressure and then introducing the moment equilibrium condition.

There have been a number of researchers working on extending the limit equilibrium method to earth pressure problems. Janbu (1957) proposed a generalized procedure to calculate earth pressure and bearing capacity, which was well documented subsequently (Janbu 1973). Chen (1975) described how slope stability, bearing capacity, and earth pressure problems can be formulated in a unified theoretical background of upper bound and lower bound analysis. Terzaghi et al. (1996) described a simplified procedure to calculate the lateral force necessary to support a vertical cut in soft clay which was believed to be applicable for φ = 0’ analysis. They used a circular slip surface passing the toe and intersecting the ground surface orthogonally. By assigning the location of the earth force resultant, which was suggested to be at the point measured 0.4 times the wall height from the bottom of the cut, they were able to calculate the active earth pressure by themoment equilibrium condition. Rahardjo and Fredlund (1984) explored the possibilities of extending the general limit equilibrium method to earth pressure problems. They used the force equilibrium condition to solve the earth force and moment equilibrium equation to locate the thrust line.

With the advent of computers, numerical technique has developed rapidly to enhance the efficiency of slope stability analysis. Chen and Morgenstern (1983) presented an analytical approach to solving the governing equations of the generalized method of slices, which would make it possible to
render an explicit formulation for earth pressure calculation. Recent developments include the automatic search technique for finding the critical slip surfaces (Baker 1980; Celestino and Duncan 1981; Li and White 1987; Chen and Shao 1988), an approach that is necessary to lead to a final solution for active earth pressures. The random search technique was developed (Chen 1992; Greco 1996) to avoid missing the global minimum factor of safety and to offer better optimization results. It appears that the method of slices has become mature enough to be extended to the field of earth pressure problems.

The governing equations

The force and moment equilibrium equations

Consider a sliding mass of a slope portrayed with either a vertical or inclined front face and divide it into a series of slices as shown in Fig. 1a. The force and moment equilibrium equations for a given slice are given by Chen and Morgenstern (1983) and by Morgenstern and Price (1965) in similar presentations, as follows:

\[ \frac{dG}{dx} - \tan \psi \frac{d\beta}{dx} G = -p(x) \sec \psi \]
\[ G \sin \beta = -y \frac{d}{dx} (G \cos \beta) + \frac{d}{dx} (y; G \cos \beta) + \eta \frac{dW}{dx} h_e \]

where

\[ p(x) = \left( \frac{dW}{dx} + q \right) \sin (\phi' - \alpha) - \left( r_u \frac{dW}{dx} \right) \sec \alpha \sin \phi' \]
\[ + \ c' \sec \alpha \cos \phi' - \eta \frac{dW}{dx} \cos (\phi' - \alpha) \]
\[ \psi = \phi' - \alpha + \beta \]

With reference to Fig. 1c, the slip surface is designated as \( y(x) \); and \( G \) and \( y \), are the magnitude and \( y \) value of the point of application of the interslice force, respectively; \( \alpha \) and \( \beta \) are the inclinations of the slice base and the interslice force, respectively; \( dW/dx \) and \( q \) are the weight of the soil mass and the vertical surface load per unit width, respectively; \( c' \) and \( \phi' \) are the effective cohesion and friction angle, respectively; \( \eta \) is the coefficient of pseudo-horizontal seismic force, which is applied at the centroid of the slice with a distance \( h_e \) to the slice base; and \( r_u \) is the coefficient of pore pressure.

We consider a general case in which a water filled tension crack is located at the crown. The boundary conditions are as follows:

\[ G(a) = P_w \]
\[ G(b) = P \]
\[ \beta(a) = 0 \]
\[ \beta(b) = \delta \]
\[ \kappa(a) = h_w/H_w = 1/3 \]
\[ h/H = \kappa(b) \]

where \( P \) is the earth pressure associated with the specified slip surface; \( H \) is the height of the wall; \( P_w \) is the water pressure at the tension crack; \( H_w \) is the height of the tension crack on which the water pressure acts; \( \delta \) is the inclination of the earth pressure with respect to the \( x \) axis; \( h \) and \( h_w \) are the distances between the point of application and the base of the slice at the wall and the tension crack, respectively; and \( \kappa \), as defined by [9] and [10], is the coefficient of the point of application.
Solutions to the differential equations

Chen and Morgenstern (1983) determined the solutions to eqs. [1] and [2] based on the boundary condition that the interslice force $G(x)$ at both ends of the failure mass is zero. For earth pressure problems where the value $G(b)$ is not zero, eqs. [1] and [2] can be solved on a more generalized basis as follows.

Solving the differential eq. [1] leads to

$$G(x) = E^{-1}(x) \left[ - \int_a^x p s \, d\xi + E(a)G(a) \right]$$

where

$$E(x) = \exp \left( - \int_a^x \tan \psi \frac{dB}{d\xi} \, d\xi \right)$$

and $\xi$ is a dummy variable substituting for $x$.

Applying the boundary conditions [5] and [6] results in eq. [14]:

$$\int_a^b p(x) s(x) \, dx = G_m$$

where

$$G_m = P_w - PE(b)$$

Integrating both parts of eq. [2], we have

$$\int_a^b G(\sin \beta - \cos \beta \tan \alpha) \, dx = \int_a^b \eta \frac{dW}{dx} \, dx + [G \cos \beta(\gamma_i - y)]_a^b$$

Defining $t(x)$ as

$$t(x) = \int_a^x (\sin \beta - \cos \beta \tan \alpha) E^{-1}(\xi) \, d\xi$$

and substituting eq. [11], the left side of [16] becomes

$$- \int_a^b (\sin \beta - \tan \alpha \cos \beta) \left[ \int_a^x p(\xi) s(\xi) \, d\xi - P_w \right] E^{-1}(x) \, dx$$

$$= \int_a^b \left[ \int_a^x p(\xi) s(\xi) \, d\xi - P_w \right] \, dx - \left[ t(x) \int_a^x p(\xi) s(\xi) \, d\xi \right]_a^b$$

$$+ \int_a^b p(x) s(x) \, dx + \int_a^b P_a \, dt$$

$$= -t(b)G_m + \int_a^b p(x) s(x) \, dx + P_a t(b)$$

$$= P[E(b)\gamma(b)] + \int_a^b P(x) s(x) \, dx$$

The right side of eq. [16] can be rewritten as

$$\int_a^b \eta \frac{dW}{dx} \, dx + h_w P_w \cos \beta_b - hP \cos \beta_b$$

Equation [16] then becomes

$$\int_a^b p(x) s(x) \gamma(x) \, dx = M_n$$

in which

$$M_n = P_w h_w - P[h \cos \delta + t(b)E(b)] + \int_a^b \eta \frac{dW}{dx} \, dx$$

The solution for earth pressure

Equations [14] and [18] involve an unknown $P$ and an unknown variable $\beta(x)$. Chen and Morgenstern (1983) suggested an assumption that defines $\beta(x)$ as shown in Fig. 1b.

$$[20] \tan \beta = f_o(x) + \kappa f(x)$$

For earth pressure problems, $f_o(x)$ can be taken as a linear variable that allows $f_o(a) = 0$ and $f_o(b) = \tan \delta$, and $f(x)$ is a sine function that is zero at $x = a$ and $x = b$ (Fig. 1b). $\delta$ is an input based on the friction at the soil-wall contact; and $h$ is also an input determined by the types of wall, as will be discussed in detail subsequently. Using eq. [20] to define tan $\beta$, $P$ and $\lambda$ can be obtained by solving eqs. [14] and [18], which can be reformulated by eliminating $P$, giving

$$[21] M_n(\lambda) = M - P_w h_w + (P_w - G) \left[ \frac{h \cos \delta}{E(b) + t(b)} \right]$$

$$- \int_a^b \eta \frac{dW}{dx} \, dx = 0$$

where

$$[22] G = \int_a^b p s \, dx$$

$$[23] M = \int_a^b p s t \, dx$$

Numerical procedures

The numerical procedures for determining the active earth pressure include the following: (1) solve eq. [21] for $\lambda$; (2) calculate $P$ from either eq. [14] or eq. [18]; and (3) for many possible slip surfaces, repeat steps (1) and (2), and locate the one that gives the maximum $P$. This value of $P$ is then taken as the solution to active earth pressure $P_a$.

The major numerical work involved in determining $P$ is the solution of eq. [21] for $\lambda$. This is best achieved by using the Newton-Raphson method.

At the iteration step $i$, the assumed value of $\lambda$, which generally gives a non-zero value of $M_n$ from eq. [21], is updated to a new value $\lambda_{i+1}$, which makes $M_n$ closer to zero by the equation

$$[24] \Delta \lambda_i = \lambda_{i+1} - \lambda_i = -M_n / \left( \frac{\partial M_n}{\partial \lambda} \right)$$

where $i = 0, 1, 2, ...$

The iterations start from an initial guess $\lambda_o$ and proceed until the following criterion is satisfied.

$$[25] |\lambda_{i+1} - \lambda_i| < \varepsilon$$

where $\varepsilon$ is a specified allowable error limit.
Fig. 2. An example of active earth pressure calculation: (a) the slip surfaces: 0, the initial estimate; 1, 2, 3, for the cases \( \kappa = 1/3, \kappa = 1/2, \) and \( \kappa = 2/3 \), respectively; (b) the earth pressures.

Equations for calculating the derivatives

It is possible to obtain an equation calculating \( \partial M / \partial \lambda \), involved in [24] by differentiating the right side of eq. [21] with respect to \( \lambda \).

\[
\frac{\partial M}{\partial \lambda} = \frac{\partial M}{\partial \lambda} + [P_w - G]
\]

where \( P_w \) is the assumed lateral load applied to the wall. The active earth pressure \( P_a \) is the value of \( P \) associated with the critical slip surface that gives \( \eta_m \), the minimum value of \( \eta \).

The numerical algorithms employed in slope stability analyses for finding the minimum factors of safety and the associated critical slip surfaces have been discussed by a number of researchers (Baker 1980; Celestino and Duncan 1981; Chen and Shao 1988; Chen 1992b). They can be readily employed in earth pressure calculations. In general, a slip surface is established by connecting \( n \) number of nodal points by a spline function. \( \eta \) can then be expressed as a function of the coordinates of the nodal points,

\[ f(x_i, y_i, x_2, y_2, ..., x_n, y_n) \]

where \( x_i, y_i, x_2, y_2, ..., x_n, y_n \) are coordinates of the nodal points. The numerical procedure starts from an initial estimate of slip surface with a loading factor \( \eta_0 \). Use of the method of optimization will enable the determination of a critical slip surface associated with \( \eta_m \).

An illustrative example

Figure 2 shows a 12 m high retaining structure backfilled with a cohesionless soil with \( \phi = 36^\circ \). The unit weight of the material is \( \gamma = 20.58 \text{kN/m}^3 \). The classical Rankine theory suggests two sets of slip line. This implies a critical slip surface inclined at an angle of \( 45^\circ + \phi/2 = 63^\circ \) to the horizontal. The active earth pressure per metre width can be calculated as follows:

\[
P_a = \frac{1}{2} \tan^2 \left( 45 - \frac{\phi}{2} \right) \gamma H^2 = 384.7 \text{kN}
\]

In the numerical analysis, we deliberately assigned \( P' = 384.7 \text{kN} \) and started with an initially estimated slip surface number 0, which was a smooth curve established by a spline function connecting three points as shown in Fig. 2a. To make it consistent with the Rankine theory, we assigned \( \delta = 0 \) and \( h = 12/3 = 4 \text{ m} \). The solutions yielded \( \eta_1 = 1.961, P = 370.05 \text{kN} \) for slip surface 0; indicating that for this slip surface, the soil mass was virtually stable and an earth pressure on a reversed direction would be needed to bring the slope into a state of limiting equilibrium. The optimization routine gave a critical slip surface of 1, which was very close to a straight line and inclined at an angle to the horizontal very close to \( (45^\circ + \phi/2) \) with the solution \( \eta = -0.006, P_a = 387.3 \text{kN} \). The value agreed well with the theoretical answer of 384.7 kN.

Conditions of physical admissibility

In conventional slope stability analyses, some restrictions to the solutions obtained from the force and moment equilibrium equations need to be enforced. These conditions become more stringent for earth pressure problems.

For a slope material with cohesion, a tension crack is generally assigned to ensure that tensile stresses are not present near the crest of the slope. Equations for calculating the depth of a tension crack can be found in Terzaghi (1943), Spencer (1973), and Janbu (1973).

It has been found that assigning a tension crack is even more important for earth pressure problems because the height of the slip surface in a retaining structure is generally small, compared to that of a natural slope. Tensile stresses will prevail over a large part of the slip surface if a tension crack is not properly specified. As a consequence, failure may occur in the iteration procedures described in the previous sections.

It has been generally accepted (Morgenstern and Price 1965) that the solution to a slope stability problem is subject to the restrictions of physical reasonability. It is required that on the interlice faces, tensile forces and shear failure are not present, i.e.,

\[
G'(x) > 0
\]

\[
G' \cos \beta' \tan \phi'_{\alpha} + c'_{\alpha} L_{\alpha} > G' \sin \beta'
\]
where the subscript ‘av’ refers to the average values on the interface, while the superscript prime refers to the effective values of their related variables. \( L \) is the height of the interface.

For earth pressure problems, a check of physical admissibility becomes even more vital, as during the search for the critical slip surface many false solutions that violate the physical requirements could possibly result. Unless these solutions are rejected, the numerical procedure will not approach to a rational solution.

### Earth pressure of different types of supports

#### Earth pressure of gravity walls

Figure 3 shows four typical examples of gravity type walls, selected from the reference Qian (1990). \( h \) is taken such that \( \kappa = h/H = 1/3 \), which is typical of gravity walls. The results are compared with those obtained by the traditional methods and are shown in Fig. 3 and Table 1. It is apparent that the numerical method presented in this paper gives results close to the classical solutions for gravity walls.

#### Earth pressure as a function of the location of point of application

Figure 2b shows the relationship between the active earth pressure and the location of the point of application, which is represented by the factor \( \kappa = h/H \). It was found that the values of \( P_a \) for \( \kappa = 1/3 \), 1/2, and 2/3 were 387.3, 599.1, and 462.8 kN, respectively. The active pressure obtained its maximum at \( \kappa = 1/2 \). The associated critical slip surface for case \( \kappa = 1/3 \) was almost a straight line (curve 1), while that for cases \( \kappa = 1/2 \) and \( \kappa = 2/3 \) (curve 2 and 3) exhibited a rather curved shape. The results are in general agreement with the understanding that a supported wall, such as a braced, anchored, or cantilever wall, which has a \( \kappa \) value close to \( 1/2 \), usually presents a larger active earth pressure, compared with that of a gravity wall. Casagrande (1973) suggested taking the coefficient of earth pressure at rest \( K_o \) as the active earth pressure coefficient for tie-back or cantilever supports. \( K_o \) was taken as \( (1 - \sin \phi') \). For this particular example, employing Casagrande’s approach will give \( K_o = 0.41 \) and \( P_a = 610.8 \) kN for \( \kappa = 1/2 \), which is close to the calculated value of 599.1 kN.

It is of practical significance to find that there exists a certain value of \( \kappa \) that gives the maximum value of \( P_a \). Although information regarding the wall deformation and pressure distributions is widely available in literature, it is not clear concerning the exact location of the point of application for a particular problem. From a practical point of view, engineers might take the maximum \( P_a \) as a conservative solution.

During the calculations for the case of \( \kappa = 1/2 \), it was found that \( P_a = 1270.0 \) kN if the restriction represented by eq. [32] was not imposed. However, after checking the normal and

<table>
<thead>
<tr>
<th>Example</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rankine</td>
<td>197.7</td>
<td>432.4</td>
<td>118.9</td>
<td>374.2</td>
</tr>
<tr>
<td>Numerical</td>
<td>191.2</td>
<td>433.0</td>
<td>120.33</td>
<td>369.0</td>
</tr>
</tbody>
</table>

Table 1. Active earth pressures of the examples shown in Fig. 3 (in kN/m).
shear force distributions for this solution, it was found that this result was unacceptable because eq. [32] was violated on the interfaces near the toe. This value would have become the final solution, replacing the accepted value of 599.1 kN, if it was not rejected. As a matter of fact, several cases have been encountered by the authors in which the false solutions were ten times as large as the one that obeys the conditions of physical admissibility. Sometimes the authors employed the random search technique (Chen 1992a) to locate the initial slip surface that obeys the physical conditions.

Discussions on earth pressures of strutted walls

The stress distribution on a strutted wall varies from one case to another depending on the nature of its supports and the stiffness of the soil material. The empirical diagrams depicting pressure distributions on a vertical strutted wall backfilled with sand, soft to medium clay, and stiff clay have been given by Terzaghi et al. (1996), from which one will obtain \( \kappa = 0.5 \) and \( \kappa = 0.44 \) for sand and clay, respectively (Fig. 4). With this information regarding the location of the point of application, we are able to compare the earth pressure calculated by the numerical method with those obtained from the empirical equations suggested by Terzaghi, Peck, and Mesri.

Following Terzaghi et al. (1967), the active earth pressure on a strutted wall backfilled with sand can be determined by the equation

\[
P_a = C_a \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) \gamma H^2
\]

where \( C_a \) is a constant equal to 0.65. Figure 5 shows the values of \( C_a \) calculated by the numerical method for \( H = 5, 10, 15, 20 \) m corresponding to \( \phi = 25^\circ \) and \( 35^\circ \). It can be seen that \( C_a \) was virtually a constant for a particular value of \( \phi \). For \( \phi = 25^\circ \), \( C_a \) was approximately equal to 0.65, while for \( \phi = 35^\circ \), the value was nearly 0.70. The numerical method essentially confirmed the Terzaghi–Peck–Mesri empirical approach but appeared to be more conservative.

With regard to strutted walls supported by soft to medium clay, Terzaghi et al. (1996) suggested the stress distribution shown in Fig. 4b. This implies that \( \kappa = 0.44 \). The active pressure coefficient \( K_a \) is determined by eq. [34], as follows:

\[
K_a = 1 - \frac{4m}{N}
\]

where

\[
N = \frac{\gamma H}{c}
\]

Terzaghi et al. suggested taking \( m = 1 \) for \( N > 4 \), and \( m < 1 \) for \( N \leq 4 \). A series of examples with \( H = 8, 11, 14, 17, 20 \) m and \( c = 30 \) and 50 kPa were performed by the numerical method. A unit weight of 18 kN/m\(^3\) was assumed. The results are plotted as \( N \) versus \( m \), shown in Fig. 6. It can be seen that a linear relationship between \( N \) and \( m \) existed, which suggests that for \( N > 6 \), adopting \( m = 1 \) is a little conservative and for \( N < 6 \), a value lower of \( m < 1 \) is necessary. More work is needed to confirm if this statement is of general significance. During the calculations, tension cracks of 3 and 5 m high for \( c = 30 \) and 50 kN, respectively, were assigned. It is of interest to note that (i) without tension cracks, the numerical method failed to offer convergent solutions for these problems that involved purely cohesive soil; and (ii) whether a tension crack was filled with water or not did not affect the results appreciably.

Discussions on earth pressures of tie-back walls

The traditional approach to earth pressure determination for tie-back walls (e.g., Canadian Geotechnical Society 1992) can be summarized by the following two steps:

1. Calculate the anchor loads \( R \) by eq. [36] for cohesionless soils or eq. [37] for clays:

\[
R = \sigma'_r A_L K_f
\]
\[ R = \alpha A_s L_s \tau_u \]

where \( \sigma_z' \) is the effective vertical stress at the midpoint of the anchor; \( A_s \) and \( L_s \) are the effective unit surface and embedded length of the anchor, respectively; \( K_f \) and \( \alpha \) are the coefficients defined in the manual; and \( \tau_u \) is the average undrained shear strength of the clay.

A factor of safety of 3 was proposed to determine the allowable loads.

(2) With the application of the allowable anchor loads, calculate the active earth pressure by the force equilibrium approaches, which implicitly assumes that \( \kappa = 1/3 \).

The pressure distribution on a tie-back wall has been investigated by a number of researchers (e.g., Hanna and Matallana 1970), and it is generally agreed that a trapezoidal shape should be more representative.

**Discussions on earth pressures of cantilever sheet-pile walls**

Design of a sheet-pile wall is based on the net pressure between the active earth pressure applied by the retained soil and the passive earth pressure provided by the restrained soil. References (Canadian Geotechnical Society 1992; Padfield and Mair 1984; King 1995) suggest both active and passive earth pressures are of triangular distribution. Determining passive earth pressure by the generalized method of slices has not been covered in this paper. However, by replacing the boundary conditions of eqs. [5] and [6] with

\[ G(a) = P_a \]
\[ G(b) = 0 \]

a similar numerical method can be developed for passive earth pressure calculations. Use of the method described in this paper for cantilever sheet-pile walls is therefore also possible.

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**Table 2. Geotechnical properties of the soil layers for the Tai Xin Square case.**

<table>
<thead>
<tr>
<th>Soil</th>
<th>Unit weight (kN/m²)</th>
<th>Friction angle ( \phi (°) )</th>
<th>Cohesion (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Fill</td>
<td>16</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>2 Sandy clay</td>
<td>18</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>3 Peat</td>
<td>15</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>4 Muck</td>
<td>18</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

In this section, we present an alternative, illustrated via a case study of a retaining structure failure. The excavation of the two-level basement of a 28-storey building at the Tai Xin Square of Zhanjiang City, China, was supported by cantilever bored piles 1.2 m in diameter at a spacing of 1.5 m (Lu et al. 1996). The geology of the soil profile consisted of 1.7–3.25 m of thick soft fill underlain by saturated plastic sandy clay, peat, and clay. The retaining piles between grid line 2 to 9 suddenly collapsed on September 25, 1994, when the excavation was 7.0–9.0 m deep (Fig. 7). This caused serious tilting of a nearby 5-storey building and the evacuation of the residents. Geotechnical parameters of various soil layers are listed in Table 2. Details are given by Lu et al. (1996). Examination of the stability of the piles consisted of the following two steps:

(1) Examination of the overall stability of the soil–pile structure by conventional slope stability analysis methods. The critical slip surface should pass through the bottom of the pile (point C in Fig. 7), and the associated minimum factor of safety should exceed an allowable limit. The examiners used Bishop’s simplified method and found a minimum factor of safety of 1.05 (Lu et al. 1996). This indicates that the overall stability was in a critical status.

Fig. 7. Failure of the excavation supported bored pole cantilever piles at the Taixin Square, Zhanjiang, China.
The generalized method of slices commonly used for slope stability analysis has been successfully extended to active earth pressure calculations. It provides a numerical approach to active earth pressure determination, applicable to problems with various stratigraphy and water levels.

(1) Implementation of this method for an earth pressure problem includes two steps: (i) Find the active pressure by solving the governing force and moment equilibrium eqs. [14] and [18]. The numerical procedures provided in this paper have allowed converged solutions for most practical problems. It has been found that assigning a tension crack is helpful in obtaining convergent solutions for a cohesive soil. (ii) Find the critical slip surface associated with the minimum load factor $\eta$. The method of optimization has proved to be effective in finding the extremes. It has been found that imposing the conditions of physical admissibility is crucial to prevent false slip surfaces from replacing the actual critical slip surface.

(2) The examples shown in this paper demonstrate the feasibility of the new method. For gravity walls, the method gave the same results as Rankine or Coulomb’s solutions. The magnitude of active earth pressure of a supported wall varies with $h$, the location of the point of application, and $P_a$ reached its maximum when $h/H$ was between 1/3 and 2/3. This is the likely location where the point of application of earth pressure for most flexible supports will lie. Comparing the numerical method with the well-established empirical approaches for strutted, tie-back, and cantilever supports confirmed that it is capable of handling problems with flexible supports. From a practical point of view, this maximum may be taken as a conservative solution to the active earth pressure of retaining walls of the types other than gravity walls.

(3) Although the results obtained by the numerical method showed its ability to evaluate the active earth pressure of different types of supports, more work is required to confirm its feasibility in handling practical problems.

It has been found that the calculated earth pressure for flexible supports in most cases is slightly greater than that evaluated by the Terzaghi–Peck–Mesri empirical criterion, and the associated critical slip surfaces exhibited a reversed curvature near the toe (see curves 2 and 3 in Fig. 2(a) and Fig. 8). Such a shape is acceptable in slope stability analysis for embankments on soft foundations. To determine whether it is acceptable in earth pressure cases requires further research work that may include collecting the evidence from case histories, investigating plastic yielding zones near the toe by model testing, and finite element analysis.

When establishing the generalized method of slices, Morgenstern and Price (1965) stated that the assumptions made for $\beta(x)$, i.e., the selection of distribution function $f(x)$, is insensitive to the final solution of the factor of safety. This conclusion was further confirmed by Whitman and Bailey (1967) and Chen and Morgenstern (1983). Investigating the sensitivity of the assumption for the distribution function, and the input value of wall friction angle $\delta$, to the solution of active pressure is a subject for further research.

References

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