Scaling Laws for UNDEX in Centrifuge

The power of shock wave at a specific distance from the epicenter of the explosion could be featured by shock wave peak pressure P_{m}, time constant θ (the interval during which shock wave pressure reduces to P_{m}/e from the peak, and e is the natural constant) [1]. While bubble energy could be characterized by the maximum bubble radius R_{bm} and period of bubble oscillation T_{b} [1] if the effects of boundaries are neglected and the bubble is assumed spherical. Values of these parameters are influenced by properties of the media, such as the atmosphere pressure P_0, fluid velocity v_1, water density ρ_{w}, and sound velocity in water c_{w}, explosive properties, i.e., explosive weight W, explosive density ρ_{e}, detonation speed v_{e}, and explosive energy Q, as well as explosion depth in water D, the distance from explosion R, and gravity acceleration ng [3], where n denotes the scale factor of gravity. In view of the Π theorem [4], shock wave and bubble parameters would be function of a set of dimensionless numbers, similar procedure of which could also be found in [2], so that

$$\begin{aligned} \frac{P_{\mathrm{m}}}{p_0} &= \mathbf{F}_1(\pi_1,\pi_2,\cdots,\pi_8) \\ \frac{\theta c_{\mathrm{w}}}{\sqrt[3]{W/\rho_{\mathrm{w}}}} &= \mathbf{F}_2(\pi_1,\pi_2,\cdots,\pi_8) \\ \frac{R_{\mathrm{bm}}}{\sqrt[3]{W/\rho_{\mathrm{w}}}} &= \mathbf{F}_3(\pi_1,\pi_2,\cdots,\pi_8) \\ \frac{T_{\mathrm{b}} c_{\mathrm{w}}}{\sqrt[3]{W/\rho_{\mathrm{w}}}} &= \mathbf{F}_4(\pi_1,\pi_2,\cdots,\pi_8) \end{aligned}, \tag{1}$$

where $\pi_1 = v_1/c_{\mathrm{w}}$ is the ratio of inertia over compressibility corresponding to the Mach number M; $\pi_2 = v_1^2/ngD$ is the ratio of inertia over gravity corresponding to the Froude number Fr; $\pi_3 = \rho_{\mathrm{w}} R^3/W$ is relevant to scaled distances; $\pi_4 = \gamma_{\mathrm{w}} nD/P_0$ relates to the effects of buoyancy; $\pi_5 = R/D$ corresponds to geometric similarity; $\pi_6 = \gamma_{\mathrm{w}} nR^4/Q$; $\pi_7 = \rho_{\mathrm{w}}/\rho_{\mathrm{e}}$; $\pi_8 = c_{\mathrm{w}}/v_{\mathrm{e}}$; and $\gamma_{\mathrm{w}} = \rho_{\mathrm{w}} g$ denotes the unit weight of water under 1g condition. Usually, water density ρ_{w} is taken as 1000kg/m^3, atmosphere pressure p_0 is 101.325kPa, and sound speed in water c_{w} is 1500m/s.

π_1 stands for the similitude of shock waves, which is determined by the compressibility of the medium. The bubble similitude is given by π_2 and π_4. It is because that the expansion, collapse, and migration of bubbles are influenced by buoyancy as well as the hydrodynamic pressure generated by gravity. To derive unified similarity for both shock wave and bubble, similitudes of Mach and Froude numbers are required, such that

$$M_{\mathrm{m}} = M_{\mathrm{p}} \Rightarrow \frac{\alpha_{\mathrm{L}}}{\alpha_{\mathrm{T}}} = 1, \tag{2}$$

$$Fr_{\mathrm{m}} = Fr_{\mathrm{p}} \Rightarrow \frac{\alpha_{\mathrm{L}}}{n\alpha_{\mathrm{T}}^2} = 1, \tag{3}$$

where subscript m stands for values of models and p stands for those of prototypes; α is a scale factor for a specific parameter denoted by the subscript between models and prototype, and subscript L, T stand for the variables of length and time, respectively. If the gravity is a constant, namely $n=1$, we obtain $\alpha_{\mathrm{L}} = \alpha_{\mathrm{T}}^2$ or $\alpha_{\mathrm{L}} = \alpha_{\mathrm{T}}$ from Eq. (2) or Eq. (3) respectively. It can be interpreted as that Eq. (2) and Eq. (3) could only be simultaneously satisfied when all the scale factors are equal to unity.

On the other hand, in a centrifuge test, if $\alpha_{\mathrm{L}} = \alpha_{\mathrm{T}} = 1/n$, scale factors satisfying both Eq. (2) and Eq. (3) could be identified. Usually, the same liquid and explosive as those in the practical prototype would be applied in the model tests. Thus, scale factor of density, stress, and velocity are all equal to unity. Consequently, scaling laws for underwater explosion in centrifuge can be derived based on dimensional analysis as summarized in Table 1.

Table 1 Scale factors for UNDEX in centrifuge

Parameters	Dimension	Scale factor	Value
Centrifugal acceleration / n	$L\,T^{-2}$	α_g	n
Explosive weight / W	M	α_W	$1/n^3$
Explosive energy / Q	$M\,L^2\,T^{-2}$	α_Q	$1/n^3$
Water depth / D Standoff distance / R Explosive radius / a_e Bubble radius / R_b Vertical migration height / h	L	α_L	$1/n$
Time constant / θ Period / T_b	T	α_T	$1/n$
Explosive Density / ρ_e Water Density / ρ_w	$M\,L^{-3}$	α_ρ	1
Atmosphere pressure / P_0 Shock wave peak pressure / P_m	$M\,L^{-1}\,T^{-2}$	α_σ	1
Fluid velocity / v_l Detonation velocity / v_e	$L\,T^{-1}$	α_v	1

Since high temperature and high pressure are generally observed during UNDEX procedure, viscidity is sufficient small to be ignored compared with the inertia. Besides, the heat energy is transferred primarily through the conduction during UNDEX procedure. Considering the relatively small coefficient of heat conduction in water of about 0.5W/mK and the short duration of an UNDEX event, the energy released from heat conduction is also negligible. Thus, scaling laws are considered to be sufficiently validated by shock wave and bubble oscillation results of UNDEX in centrifuge.

In all centrifuge tests, π_1 and π_2, which represent the respective Mach and Froude numbers, are also same for the models and the porotypes. And π_5, denoting geometric similitude, would also be met in model tests. If the scope of research is focused on a specific explosive and the same liquid is used, π_7, π_8 would be constant, and π_6 is then consistent with π_3 as specific energy is identical for models and prototypes. For simplicity, the similitudes of π_1 and π_2 could be indirectly described by more easily calculated π_3 and π_4, which are only related to shock wave or bubble characteristics respectively.

According to Cole [1], when shock wave peak pressure is considerably higher compared with the hydrodynamic pressure, the water depth D would have no significant influence on shock wave pressure, and π_2 or π_4 can therefore be neglected. Consequently, for shock wave, Eq. (1) can then be simplified as

$$P_m / P_0 = \mathbf{F}_1(\pi_3) = \mathbf{F}_1(\rho_w R^3 / W), \tag{4}$$

$$\theta c_w / \sqrt[3]{W / \rho_w} = \mathbf{F}_2(\pi_3) = \mathbf{F}_2(\rho_w R^3 / W), \tag{5}$$

Moreover, radius and period are inherent characteristics of bubbles under certain hydrodynamic pressure, which are independent of measuring distance and fluid compressibility [1]. Thus, π_1 and π_3 could be neglected. Similarly, for bubble oscillation

$$R_{bm} / \sqrt[3]{W / \rho_w} = \mathbf{F}_3(\pi_4) = \mathbf{F}_3(\gamma_w nD / P_0), \tag{6}$$

$$T_b c_w / \sqrt[3]{W / \rho_w} = \mathbf{F}_4(\pi_4) = \mathbf{F}_4(\gamma_w nD / P_0), \tag{7}$$

The testing scheme for validating Eq. (4) to Eq. (7) is organized as follows:

(1) Arrange a group of centrifuge tests having the same π_3 to investigate if the testing results can be regressed to the unique functions of $\mathbf{F}_1$ and $\mathbf{F}_2$ as represented by Eq. (4) and Eq. (5). The positive outcomes will serve as a strong evidence on scaling laws as listed in Table 1 and the applicability of centrifuge modeling on underwater explosion. This statement is referred to as the “scaling laws validation” phase in the present study.

(2) Organizing a group of centrifuge tests having different π_3 to derive the empirical co-relationship for $\mathbf{F}_1$ and $\mathbf{F}_2$. These relationships are readily available as proposed by a number of researches and our work is focused on validating them in terms of the formulations and the relevant empirical coefficients. This is statement referred to as the “formulation and calibration” phase in the present study.

(3) Performing the similar phases for bubble oscillation, i.e., investigating the group of centrifuge tests having the same π_4 for theoretical validation purposes and having different π_4 for calibration on $\mathbf{F}_3$ and $\mathbf{F}_4$.

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